

# A ONE STEP NUMERICAL METHOD FOR THE SOLUTION OF INITIAL VALUE PROBLEMS

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## Abstract

In this paper we develop a one-step numerical method based on logarithm, exponential, trigonometric and hyperbolic (LETH) basis functions which is used for the solution of initial value problems. The method is used to solve some selected initial value problems numerically, the results obtained shows that the method is reliable computationally.

**Keywords:** Ordinary Differential Equations, Initial Value Problem, Basis Functions, Lipschitz Constant, Logarithm, Exponential, Trigonometric, Hyperbolic.



## 1. INTRODUCTION

Many physical phenomena in engineering, physical, life, social, biological and medical sciences are modeled by differential equations. It is a known fact that many of these equations arising from real life situations usually are not amenable to analytical techniques, therefore the need for numerical methods. Many numerical methods have been developed to solve differential equations, from one step methods, linear multistep methods, block and hybrid methods Odekunle et al, (2012), Rosser, (1967). Fatunla, (1976, 1988) and Ibijola et al, (2011) worked on numerical one step method which can effectively cope with the IVPs with oscillatory and exponential solutions. Based on this we have developed a one-step numerical method that is based on the combination of logarithm, exponential, trigonometric and hyperbolic (LETH) functions respectively, which is well suited for problems involving Logarithm, exponential and trigonometric solutions. This will be used to find numerical solution to selected initial value problems of the form,

$$y' = f(x, y), y(a) = y_0 \quad (1)$$

The outline of this paper is in the following order; in section one, we have the introduction, we state some definitions and existing theorems, section two is the derivation of the new numerical method, in section three we have implementation, section four is discussion of results, section five is conclusion. We assume that

equation (1) satisfies the existence and uniqueness theorem stated below

**Theorem 1** Fatunla, (1988), Lambert, (1973, 1991)

Let  $f(x, y)$  be defined and continuous for all points  $(x, y)$  in the region  $D$  defined by  $\{(x, y): a \leq x \leq b, -\infty < y < \infty\}$  where  $a$  and  $b$  are finite, and let there exist a constant  $L$  such that for every  $x, y, y^*$  such that  $(x, y)$  and  $(x, y^*)$  are both in  $D$ . Then

$$|f(x, y) - f(x, y^*)| \leq L|y - y^*| \quad (2)$$

If  $y_0$  is any given number there exist a unique solution  $y(x)$  of the initial value problem (1). The inequality (2) is known as the Lipschitz condition and the constant  $L$  is called the Lipschitz constant.

**Theorem 2** Lambert, (1973): Let  $f(x, y)$  be defined and continuous in region  $D$  defined above and in addition, satisfy equation (2). Then the initial value problem (1) has a unique solution in  $D$ .

**Definition 1** Fatunla , (1988), Henrici (1962): If the initial value problem (1) satisfies the existence and uniqueness theorem (Theorem 2), then the increment function denoted by  $\phi(x_n, y_n; h)$  satisfies the Lipschitz condition of order one with respect to the dependent variable  $y$ .

## 2. DERIVATION OF THE ONE STEP METHOD

In this section we consider a basis function consisting of logarithm, exponential, trigonometric and hyperbolic (LETH) functions of the form:

$$F(x) = a_1 e^{\sec(x)} + a_2 (3^{x+1} + 3) + a_3 (\cosh(x) + \sin(x)) \quad (3)$$

where  $a_1, a_2$  and  $a_3$  are real undetermined coefficients. Assume that the theoretical solution  $y(x)$  to initial value problem (1) can be locally represented in the interval  $[x_n, x_{n+1}]$  where  $n \geq 0$  by (3).

We assume that  $y_n$  approximates  $y(x_n)$  and that  $f_n = f(x_n, y_n)$ , we also define grid points as  $x_n = a + nh$ , where  $n = 0, 1, 2, 3, \dots$  (4)

Imposing these constraint on the function above so that the function coincides with the theoretical solution at points

$x = x_n$  and  $x = x_{n+1}$  i.e.  $F(x_n)$  and  $F(x_{n+1})$  coincides with  $y_n$  and  $y_{n+1}$  respectively so that

$$\begin{aligned} F(x_n) &= a_1 e^{\sec(x_n)} + a_2 (3^{x_{n+1}} + 3) + \\ &a_3 (\cosh(x_n) + \sin(x_n)) = y_n \end{aligned} \quad (5)$$

$$\text{and } F(x_{n+1}) = a_1 e^{\sec(x_{n+1})} + a_2 (3^{x_{n+1}+1} + 3) + a_3 (\cosh(x_{n+1}) + \sin(x_{n+1})) = y_{n+1} \quad (6)$$

We require the derivatives of the above function to coincides with the differential equation as well as its first, second, third and fourth derivatives with respect to  $x$  at  $x = x_n$  such that

$$F^1(x_n) = f_n$$

$$F^2(x_n) = f_n^1$$

$$F^3(x_n) = f_n^2$$

(7)

Which implies that

$$\begin{aligned} F^1(x_n) &= a_1 * \sec(x_n) * \tan(x_n) * \exp(\sec(x_n)) + \\ &a_2 * 3^{(x_{n+1})} * \ln(3) + a_3 * (\sinh(x_n) + \cos(x_n)) = f_0 \end{aligned} \quad (8)$$

$$\begin{aligned} F^2(x_n) &= a_1 * \sec(x_n) * \tan^2(x_n) * \exp(\sec(x_n)) + \\ &a_1 * \sec(x_n) * (1 + \tan^2(x_n)) * \exp(\sec(x_n)) + a_1 * \\ &\sec^2(x_n) * \tan^2(x_n) * \exp(\sec(x_n)) + a_2 * 3^{(x_{n+1})} * \end{aligned}$$

$$\begin{aligned} \ln(3)^2 + a_3 * (\cosh(x_n) - \sin(x_n)) &= f_1 \\ & \end{aligned} \quad (9)$$

$$\begin{aligned} F^3(x_n) &= a_1 * \sec(x_n) * \tan^3(x_n) * \exp(\sec(x_n)) + \\ &5 * a_1 * \sec(x_n) * \tan(x_n) * \exp(\sec(x_n)) * (1 + \\ &\tan^2(x_n)) + 3 * a_1 * \sec^2(x_n) * \tan^3(x_n) * \\ &\exp(\sec(x_n)) + 3 * a_1 * \sec^2(x_n) * (1 + \tan^2(x_n)) * \\ &\tan(x_n) * \exp(\sec(x_n)) + a_1 * \sec^3(x_n) * \tan^3(x_n) * \\ &\exp(\sec(x_n)) + a_2 * 3^{(x_{n+1})} * \ln(3)^3 + a_3 * \\ &(\sinh(x_n) - \cos(x_n)) = f_2 \end{aligned} \quad (10)$$

Solving equations (8), (9) and (10) simultaneously using Maple, we obtain the following values for the coefficients  $a_1, a_2$  and  $a_3$  as follows:

$$\begin{aligned} a_1 &= \frac{-(\cosh(x_n) * \ln(3)^2 * f_0 - \sin(x_n) * \ln(3)^2 * f_0 \\ &- \ln(3)^2 * \sinh(x_n) * f_1 - \ln(3)^2 * \cos(x_n) * f_1 - \ln(3) \\ &* \sinh(x_n) * f_0 + \ln(3) * \sinh(x_n) * f_2 + \ln(3) * \cos(x_n) \\ &* f_0 + \ln(3) * \cos(x_n) * f_2 - \cosh(x_n) * f_2 \\ &+ \sin(x_n) * f_2 + f_1 * \sinh(x_n) - f_1 * \cos(x_n))}{(-\sec^2(x_n) * \tan^3(x_n) * \ln(3) * \sinh(x_n) - \sec^2(x_n) \\ * \tan^3(x_n) * \ln(3) * \cos(x_n) - 6 * \sec(x_n) * \tan^3(x_n) * \ln(3) \\ * \sinh(x_n) - 6 * \sec(x_n) * \tan^3(x_n) * \ln(3) * \cos(x_n) + \\ \sec(x_n) * \tan^2(x_n) * \ln(3)^2 * \sinh(x_n) + \sec(x_n) * \tan^2(x_n) \\ * \ln(3)^2 * \cos(x_n) - 3 * \sec(x_n) * \tan(x_n) * \ln(3) * \sinh(x_n) \\ - 3 * \sec(x_n) * \tan(x_n) * \ln(3) * \cos(x_n) + 6 * \cosh(x_n) * \\ \tan^3(x_n) - 6 * \sin(x_n) * \tan^3(x_n) - 2 * \tan^2(x_n) * \sinh(x_n) \\ + 2 * \tan^2(x_n) * \cos(x_n) + 5 * \cosh(x_n) * \tan(x_n) - 5 * \sin(x_n) \\ * \tan(x_n) + \ln(3)^2 * \sinh(x_n) + \ln(3)^2 * \cos(x_n) + \cos(x_n) \\ - \sinh(x_n) + \cosh(x_n) * \sec^2(x_n) * \tan^3(x_n) - \cosh(x_n) * \\ \tan(x_n) * \ln(3)^2 + \sec(x_n) * \tan^2(x_n) * \cos(x_n) + \sin(x_n) \\ * \tan(x_n) * \ln(3)^2 + 6 * \cosh(x_n) * \sec(x_n) * \tan^3(x_n) - \\ 6 * \tan(x_n) * \ln(3) * \cos(x_n) - 4 * \tan(x_n) * \ln(3) * \sinh(x_n) \\ - 3 * \sin(x_n) * \sec(x_n) * \tan(x_n) - \sin(x_n) * \sec^2(x_n) * \\ \tan^3(x_n) + 2 * \tan^2(x_n) * \ln(3)^2 * \cos(x_n) - 6 * \tan^3(x_n) \\ * \ln(3) * \cos(x_n) - 6 * \tan^3(x_n) * \ln(3) * \sinh(x_n) - \sec(x_n) \\ * \tan^2(x_n) * \sinh(x_n) + 3 * \cosh(x) * \sec(x_n) * \tan(x_n) \\ + 2 * \tan^2(x_n) * \ln(3)^2 * \sinh(x_n) - 6 * \sin(x_n) * \sec(x_n) \\ * \tan^3(x_n)) * \sec(x_n) * \exp(\sec(x_n)))} \\ &= \end{aligned} \quad (11)$$

$$\begin{aligned}
 & a_2 \\
 & (\cosh(x) * \sec^2(x_n) * \tan^3(x_n) * f0 - \sin(x) \\
 & * \sec^2(x_n) * \tan^3(x_n) * f0 - \sec^2(x_n) * \tan^3(x_n) \\
 & * \sinh(x) * f1 - \sec^2(x_n) * \tan^3(x_n) * \cos(x) \\
 & * f1 + 6 * \cosh(x) * \sec(x) * \tan^3(x_n) * f0 \\
 & - 6 * \sin(x) * \sec(x) * \tan^3(x_n) * f0 - \\
 & 6 * \sec(x) * \tan^3(x_n) * \sinh(x) * f1 \\
 & - 6 * \sec(x) * \tan^3(x_n) * \cos(x) * f1 \\
 & - \sec(x) * \tan(x)^2 * \sinh(x) * f0 + \sec(x) \\
 & * \tan(x)^2 * \cos(x) * f0 + \sec(x) * \tan(x)^2 \\
 & * \sinh(x) * f2 + f2 * \cos(x) + f2 * \sinh(x) \\
 & + f0 * \cos(x) - f0 * \sinh(x) - 4 * \tan(x) \\
 & * \sinh(x) * f1 - 6 * \tan(x) * \cos(x) * f1 \\
 & + 2 * \tan(x)^2 * \cos(x) * f2 + 5 * \cosh(x) * \tan(x) \\
 & * f0 - \cosh(x) * \tan(x) * f2 + \sin(x) * \tan(x) * \\
 & f2 - 5 * \sin(x) * \tan(x) * f0 - 6 * \tan(x)^3 * \cos(x) \\
 & * f1 - 2 * \tan(x)^2 * \sinh(x) * f0 + 2 * \tan(x)^2 \\
 & * \sinh(x) * f2 + 2 * \tan(x)^2 * \cos(x) * f0 + 6 \\
 & * \cosh(x) * \tan(x)^3 * f0 - 6 * \tan(x)^3 * \sinh(x) \\
 & * f1 - 6 * \sin(x) * \tan^3(x) * f0 + 3 * \cosh(x) \\
 & * \sec(x) * \tan(x) * f0 + \sec(x) * \tan(x)^2 * \cos(x) \\
 & * f2 - 3 * \sin(x) * \sec(x) * \tan(x) * f0 - 3 * \sec(x) * \\
 & \tan(x) * \sinh(x) * f1 - 3 * \sec(x) * \tan(x) * \cos(x) * f1) \\
 = & \frac{((-\sec^2(x_n) * \tan^3(x_n) * \ln(3) * \sinh(x_n) - \sec^2(x_n) \\
 & * \tan^3(x_n) * \ln(3) * \cos(x_n) - 6 * \sec(x_n) * \tan^3(x_n) * \ln(3) \\
 & * \sinh(x_n) - 6 * \sec(x_n) * \tan^3(x_n) * \ln(3) * \cos(x_n) + \\
 & \sec(x_n) * \tan^2(x_n) * \ln(3)^2 * \sinh(x_n) + \sec(x_n) * \tan^2(x_n) \\
 & * \ln(3)^2 * \cos(x_n) - 3 * \sec(x_n) * \tan(x_n) * \ln(3) * \sinh(x_n) \\
 & - 3 * \sec(x_n) * \tan(x_n) * \ln(3) * \cos(x_n) + 6 * \cosh(x_n) * \\
 & \tan^3(x_n) - 6 * \sin(x_n) * \tan^3(x_n) - 2 * \tan^2(x_n) * \sinh(x_n) \\
 & + 2 * \tan^2(x_n) * \cos(x_n) + 5 * \cosh(x_n) * \tan(x_n) - 5 * \sin(x_n) \\
 & * \tan(x_n) + \ln(3)^2 * \sinh(x_n) + \ln(3)^2 * \cos(x_n) + \cos(x_n) \\
 & - \sinh(x_n) + \cosh(x_n) * \sec^2(x_n) * \tan^3(x_n) - \cosh(x_n) * \\
 & \tan(x_n) * \ln(3)^2 + \sec(x_n) * \tan^2(x_n) * \cos(x_n) + \sin(x_n) \\
 & * \tan(x_n) * \ln(3)^2 + 6 * \cosh(x_n) * \sec(x_n) * \tan^3(x_n) - \\
 & 6 * \tan(x_n) * \ln(3) * \cos(x_n) - 4 * \tan(x_n) * \ln(3) * \sinh(x_n) \\
 & - 3 * \sin(x_n) * \sec(x_n) * \tan(x_n) - \sin(x_n) * \sec^2(x_n) * \\
 & \tan^3(x_n) + 2 * \tan^2(x_n) * \ln(3)^2 * \cos(x_n) - 6 * \tan^3(x_n) \\
 & * \ln(3) * \cos(x_n) - 6 * \tan^3(x_n) * \ln(3) * \sinh(x_n) - \sec(x_n) \\
 & * \tan^2(x_n) * \sinh(x_n) + 3 * \cosh(x) * \sec(x_n) * \tan(x_n) \\
 & + 2 * \tan^2(x_n) * \ln(3)^2 * \sinh(x_n) - 6 * \sin(x_n) * \sec(x_n) \\
 & * \tan^3(x_n)) * \sec(x_n) * \exp(\sec(x_n)))}{(-\sec^2(x_n) * \tan^3(x_n) * \ln(3) * \sinh(x_n) - \sec^2(x_n) \\
 & * \tan^3(x_n) * \ln(3) * \cos(x_n) - 6 * \sec(x_n) * \tan^3(x_n) * \ln(3) \\
 & * \sinh(x_n) - 6 * \sec(x_n) * \tan^3(x_n) * \ln(3) * \cos(x_n) + \\
 & \sec(x_n) * \tan^2(x_n) * \ln(3)^2 * \sinh(x_n) + \sec(x_n) * \tan^2(x_n) \\
 & * \ln(3)^2 * \cos(x_n) - 3 * \sec(x_n) * \tan(x_n) * \ln(3) * \sinh(x_n) \\
 & - 3 * \sec(x_n) * \tan(x_n) * \ln(3) * \cos(x_n) + 6 * \cosh(x_n) * \\
 & \tan^3(x_n) - 6 * \sin(x_n) * \tan^3(x_n) - 2 * \tan^2(x_n) * \sinh(x_n) \\
 & + 2 * \tan^2(x_n) * \cos(x_n) + 5 * \cosh(x_n) * \tan(x_n) - 5 * \sin(x_n) \\
 & * \tan(x_n) + \ln(3)^2 * \sinh(x_n) + \ln(3)^2 * \cos(x_n) + \cos(x_n) \\
 & - \sinh(x_n) + \cosh(x_n) * \sec^2(x_n) * \tan^3(x_n) - \cosh(x_n) * \\
 & \tan(x_n) * \ln(3)^2 + \sec(x_n) * \tan^2(x_n) * \cos(x_n) + \sin(x_n) \\
 & * \tan(x_n) * \ln(3)^2 + 6 * \cosh(x_n) * \sec(x_n) * \tan^3(x_n) - \\
 & 6 * \tan(x_n) * \ln(3) * \cos(x_n) - 4 * \tan(x_n) * \ln(3) * \sinh(x_n) \\
 & - 3 * \sin(x_n) * \sec(x_n) * \tan(x_n) - \sin(x_n) * \sec^2(x_n) * \\
 & \tan^3(x_n) + 2 * \tan^2(x_n) * \ln(3)^2 * \cos(x_n) - 6 * \tan^3(x_n) \\
 & * \ln(3) * \cos(x_n) - 6 * \tan^3(x_n) * \ln(3) * \sinh(x_n) - \sec(x_n) \\
 & * \tan^2(x_n) * \sinh(x_n) + 3 * \cosh(x) * \sec(x_n) * \tan(x_n) \\
 & + 2 * \tan^2(x_n) * \ln(3)^2 * \sinh(x_n) - 6 * \sin(x_n) * \sec(x_n) \\
 & * \tan^3(x_n)) * \sec(x_n) * \exp(\sec(x_n)))} \quad (13)
 \end{aligned}$$

(12)

Imposing the assumption  $F(x_n) = y_n$  and  $F(x_{n+1}) = y_{n+1}$  on equations (5) and (6) and substituting the values of the coefficients  $a_1$ ,  $a_2$  and  $a_3$  we have

$$y_{n+1} - y_n = F(x_{n+1}) - F(x_n) \quad (14)$$

which gives us

$$y_{n+1} - y_n = a_1(e^{\sec(x_{n+1})} - e^{\sec(x_n)}) + a_2(3^{x_{n+1}+1} - 3^{x_n+1}) + a_3(\cosh(x_{n+1}) + \sin(x_{n+1}) - \cosh(x_n) - \sin(x_n)) \quad (15)$$

Using equation (4) we can rewrite equation (15) in a simplified form as

$$y_{n+1} - y_n = a_1 * \left( \exp\left(\frac{1}{\cos(x_n) * \cos(h) - \sin(x_n) * \sin(h)}\right) - \exp\left(\frac{1}{\cos(x_n)}\right) + a_2 * 3^{(x_n+1)} * (3^{(h-1)}) + a_3 * (\cosh(x_n) * \cosh(h) + \sinh(x_n) * \sinh(h) + \sin(x_n) * \cos(h) + \cos(x_n) * \sin(h) - \cosh(x_n)) - \sin(x_n)) + a_3((\cosh(h) - 1)(\cosh(x_n) + \sin(x_n) + \sinh(x_n) * \sinh(h) + \cos(x_n) * \sin(h)) \right)$$

(16)

Substituting the values of coefficients  $a_1$ ,  $a_2$  and  $a_3$  from equations (11), (12) and (13) respectively we have

$$\begin{aligned} y_{n+1} &= y_n + \left( \begin{array}{l} \left( -(\cosh(x_n) * \ln(3)^2 * f0 - \sin(x_n) * \ln(3)^2 * f0 - \ln(3)^2 * \sinh(x_n) * f1 \right. \right. \\ \left. \left. - \ln(3)^2 * \cos(x_n) * f1 - \ln(3) * \sinh(x_n) * f0 + \ln(3) * \sinh(x_n) * f2 + \ln(3) * \cos(x_n) * f0 \right. \right. \\ \left. \left. + \ln(3) * \cos(x_n) * f2 - \cosh(x_n) * f2 + \sin(x_n) * f2 + f1 * \sinh(x_n) - f1 * \cos(x_n) \right) \right. \\ \left. \left. * \left( \exp\left(\frac{1}{\cos(x_n) * \cos(h) - \sin(x_n) * \sin(h)}\right) - \exp\left(\frac{1}{\cos(x_n)}\right) \right) \right. \right. \\ \left. \left. \left( (-6 * \tan(x_n) * \ln(3) * \cos(x_n) + \sec(x_n) * \tan(x_n)^2 * \cos(x_n) - \cosh(x_n) * \tan(x_n) * \ln(3)^2 \right. \right. \right. \\ \left. \left. \left. - 6 * \tan(x_n)^3 * \ln(3) * \sinh(x_n) - \sec(x_n) * \tan(x_n)^2 * \sinh(x_n) + 3 * \cosh(x_n) * \sec(x_n) * \tan(x_n) \right. \right. \right. \\ \left. \left. \left. - 4 * \tan(x_n) * \ln(3) * \sinh(x_n) + 6 * \cosh(x_n) * \sec(x_n) * \tan(x_n)^3 - 6 * \sin(x_n) * \sec(x_n) * \tan(x_n)^3 \right. \right. \right. \\ \left. \left. \left. + 2 * \tan(x_n)^2 * \ln(3)^2 * \sinh(x_n) + \sin(x_n) * \tan(x_n) * \ln(3)^2 - 3 * \sin(x_n) * \sec(x_n) * \tan(x_n) \right. \right. \right. \\ \left. \left. \left. + \cosh(x_n) * \sec(x_n)^2 * \tan(x_n)^3 + 2 * \tan(x_n)^2 * \ln(3)^2 * \cos(x_n) - 6 * \tan(x_n)^3 * \ln(3) * \cos(x_n) \right. \right. \right. \\ \left. \left. \left. - \sin(x_n) * \sec(x_n)^2 * \tan(x_n)^3 - \sec(x_n)^2 * \tan(x_n)^3 * \ln(3) * \sinh(x_n) \right. \right. \right. \\ \left. \left. \left. - \sec(x_n)^2 * \tan(x_n)^3 * \ln(3) * \cos(x_n) - 6 * \sec(x_n) * \tan(x_n)^3 * \ln(3) * \sinh(x_n) \right. \right. \right. \\ \left. \left. \left. - 6 * \sec(x_n) * \tan(x_n)^3 * \ln(3) * \cos(x_n) + \sec(x_n) * \tan(x_n)^2 * \ln(3)^2 * \sinh(x_n) \right. \right. \right. \\ \left. \left. \left. + \sec(x_n) * \tan(x_n)^2 * \ln(3)^2 * \cos(x_n) - 3 * \sec(x_n) * \tan(x_n) * \ln(3) * \sinh(x_n) \right. \right. \right. \\ \left. \left. \left. - 3 * \sec(x_n) * \tan(x_n) * \ln(3) * \cos(x_n) + 5 * \cosh(x_n) * \tan(x_n) \right. \right. \right. \\ \left. \left. \left. - 5 * \sin(x_n) * \tan(x_n) + \ln(3)^2 * \sinh(x_n) + \ln(3)^2 * \cos(x_n) + 6 * \cosh(x_n) * \tan(x_n)^3 \right. \right. \right. \\ \left. \left. \left. - \sinh(x_n) + \cos(x_n) - 6 * \sin(x_n) * \tan(x_n)^3 - 2 * \tan(x_n)^2 * \sinh(x_n) + 2 * \tan(x_n)^2 * \cos(x_n) \right. \right. \right. \\ \left. \left. \left. * \sec(x_n) * \exp(\sec(x_n)) \right) \right. \right. \end{array} \right) \end{aligned}$$

$$\begin{aligned} & \left( f0 * \cos(x_n) + f2 * \cos(x_n) - f0 * \sinh(x_n) + f2 * \sinh(x_n) - 6 * \sec(x_n) * \tan(x_n)^3 * \sinh(x_n) * f1 \right. \\ & \quad - 6 * \sec(x_n) * \tan(x_n)^3 * \cos(x_n) * f1 + 6 * \cosh(x_n) * \sec(x) * \tan(x_n)^3 * f0 \\ & \quad - 6 * \sin(x_n) * \sec(x_n) * \tan(x_n)^3 * f0 - \sec(x_n) * \tan(x_n)^2 * \tan(x_n)^3 * \sinh(x_n) * f1 - \sec(x_n)^2 \\ & \quad * \tan(x_n)^3 * \cos(x_n) * f1 + \cosh(x_n) * \sec(x_n)^2 * \tan(x_n)^3 * f0 - \sin(x_n) * \sec(x_n)^2 \\ & \quad * \tan(x_n)^3 * f0 + \sin(x_n) * \tan(x_n) * f2 - 6 * \tan(x_n) * \cos(x_n) * f1 - 4 * \tan(x_n) * \sinh(x_n) * f1 \\ & - 5 * \sin(x_n) * \tan(x_n) * f0 + 5 * \cosh(x_n) * \tan(x_n) * f0 - \cosh(x_n) * \tan(x_n) * f2 + 6 * \cosh(x_n) * \tan(x_n)^3 * f0 \\ & \quad - 6 * \tan(x_n)^3 * \sinh(x_n) * f1 - 6 * \tan(x_n)^3 * \cos(x_n) * f1 - 2 * \tan(x_n)^2 * \sinh(x_n) * f0 \\ & \quad + 2 * \tan(x_n)^2 * \cos(x_n) * f2 + 2 * \tan(x_n)^2 * \sinh(x_n) * f2 + 2 * \tan(x_n)^2 * \cos(x_n) * f0 - 6 * \sin(x_n) \\ & \quad * \tan(x_n)^3 * f0 - \sec(x_n) * \tan(x_n)^2 * \sinh(x_n) * f0 + \sec(x_n) * \tan(x_n)^2 * \sinh(x_n) * f2 \\ & + \sec(x_n) * \tan(x_n)^2 * \cos(x_n) * f0 + \sec(x_n) * \tan(x_n)^2 * \cos(x_n) * f2 + 3 * \cosh(x_n) * \sec(x_n) * \tan(x_n) * f0 \\ & - 3 * \sin(x_n) * \sec(x_n) * \tan(x_n) * f0 - 3 * \sec(x_n) * \tan(x_n) * \sinh(x_n) * f1 - 3 * \sec(x_n) * \tan(x_n) * \cos(x_n) * f1 * \\ & \quad (3^h - 1) \\ + & \frac{\left( (-6 * \tan(x_n) * \ln(3) * \cos(x_n) + \sec(x_n) * \tan(x_n)^2 * \cos(x_n) - \cosh(x_n) * \tan(x_n) * \ln(3)^2 \right.}{\left( (-6 * \tan(x_n)^3 * \ln(3) * \sinh(x_n) - \sec(x_n) * \tan(x_n)^2 * \sinh(x_n) + 3 * \cosh(x_n) * \sec(x_n) * \tan(x_n) \right.} \\ & \quad - 6 * \tan(x_n)^3 * \ln(3) * \sinh(x_n) - \sec(x_n) * \tan(x_n)^2 * \sinh(x_n) + 3 * \cosh(x_n) * \sec(x_n) * \tan(x_n)^3 \\ & \quad - 4 * \tan(x_n) * \ln(3) * \sinh(x_n) + 6 * \cosh(x_n) * \sec(x_n) * \tan(x_n)^3 - 6 * \sin(x_n) * \sec(x_n) * \tan(x_n)^3 \\ & \quad + 2 * \tan(x_n)^2 * \ln(3)^2 * \sinh(x_n) + \sin(x_n) * \tan(x_n) * \ln(3)^2 - 3 * \sin(x_n) * \sec(x_n) * \tan(x_n) \\ & \quad + \cosh(x_n) * \sec(x_n)^2 * \tan(x_n)^3 + 2 * \tan(x_n)^2 * \ln(3)^2 * \cos(x_n) - 6 * \tan(x_n)^3 * \ln(3) * \cos(x_n) \\ & \quad - \sin(x_n) * \sec(x_n)^2 * \tan(x_n)^3 - \sec(x_n)^2 * \tan(x_n)^3 * \ln(3) * \sinh(x_n) \\ & \quad - \sec(x_n)^2 * \tan(x_n)^3 * \ln(3) * \cos(x_n) - 6 * \sec(x_n) * \tan(x_n)^3 * \ln(3) * \sinh(x_n) \\ & \quad - 6 * \sec(x_n) * \tan(x_n)^3 * \ln(3) * \cos(x_n) + \sec(x_n) * \tan(x_n)^2 * \ln(3)^2 * \sinh(x_n) \\ & \quad + \sec(x_n) * \tan(x_n)^2 * \ln(3)^2 * \cos(x_n) - 3 * \sec(x_n) * \tan(x_n) * \ln(3) * \sinh(x_n) \\ & \quad - 3 * \sec(x_n) * \tan(x_n) * \ln(3) * \cos(x_n) + 5 * \cosh(x_n) * \tan(x_n) \\ & \quad - 5 * \sin(x_n) * \tan(x_n) + \ln(3)^2 * \sinh(x_n) + \ln(3)^2 * \cos(x_n) + 6 * \cosh(x_n) * \tan(x_n)^3 \\ & \quad - \sinh(x_n) + \cos(x_n) - 6 * \sin(x_n) * \tan(x_n)^3 - 2 * \tan(x_n)^2 * \sinh(x_n) + 2 * \tan(x_n)^2 * \cos(x_n)) \\ & \quad * \ln(3)) \end{aligned}$$

$$\begin{aligned}
 & -(\sec(x_n)^2 * \tan(x_n)^3 * \ln(3) * f0 - \sec(x_n)^2 * \tan(x_n)^3 * f1 + 6 * \sec(x_n) * \tan(x_n)^3 * \ln(3) * f0 \\
 & -\sec(x_n) * \tan(x_n)^2 * \ln(3)^2 * f0 - 6 * \sec(x_n) * \tan(x_n)^3 * f1 + 6 * \tan(x_n)^3 * \ln(3) * f0 \\
 & -2 * \tan(x_n)^2 * \ln(3)^2 * f0 + \sec(x_n) * \tan(x_n)^2 * f2 + 3 * \sec(x_n) * \tan(x_n) * \ln(3) * f0 \\
 & -6 * \tan(x_n)^3 * f1 + \tan(x_n) * \ln(3)^2 * f1 - 3 * \sec(x_n) * \tan(x_n) * f1 + 2 * \tan(x_n)^2 * f2 \\
 & + 5 * \tan(x_n) * \ln(3) * f0 - \tan(x_n) * \ln(3) * f2 - \ln(3)^2 * f0 - 5 * \tan(x_n) * f1 + f2) * \\
 & * ((\cosh(h) - 1) * \cosh(x_n) + (\cos(h) - 1) * \sinh(x_n) * \sinh(h) + \cos(x_n) * \sin(h)) \\
 & + \frac{((-6 * \tan(x_n) * \ln(3) * \cos(x_n) + \sec(x_n) * \tan(x_n)^2 * \cos(x_n) - \cosh(x_n) * \tan(x_n) * \ln(3)^2 \\
 & -6 * \tan(x_n)^3 * \ln(3) * \sinh(x_n) - \sec(x_n) * \tan(x_n)^2 * \sinh(x_n) + 3 * \cosh(x_n) * \sec(x_n) * \tan(x_n) \\
 & -4 * \tan(x_n) * \ln(3) * \sinh(x_n) + 6 * \cosh(x_n) * \sec(x_n) * \tan(x_n)^3 - 6 * \sin(x_n) * \sec(x_n) * \tan(x_n)^3 \\
 & + 2 * \tan(x_n)^2 * \ln(3)^2 * \sinh(x_n) + \sin(x_n) * \tan(x_n) * \ln(3)^2 - 3 * \sin(x_n) * \sec(x_n) * \tan(x_n) \\
 & + \cosh(x_n) * \sec(x_n)^2 * \tan(x_n)^3 + 2 * \tan(x_n)^2 * \ln(3)^2 * \cos(x_n) - 6 * \tan(x_n)^3 * \ln(3) * \cos(x_n) \\
 & - \sin(x_n) * \sec(x_n)^2 * \tan(x_n)^3 - \sec(x_n)^2 * \tan(x_n)^3 * \ln(3) * \sinh(x_n) \\
 & - \sec(x_n)^2 * \tan(x_n)^3 * \ln(3) * \cos(x_n) - 6 * \sec(x_n) * \tan(x_n)^3 * \ln(3) * \sinh(x_n) \\
 & - 6 * \sec(x_n) * \tan(x_n)^3 * \ln(3) * \cos(x_n) + \sec(x_n) * \tan(x_n)^2 * \ln(3)^2 * \sinh(x_n) \\
 & + \sec(x_n) * \tan(x_n)^2 * \ln(3)^2 * \cos(x_n) - 3 * \sec(x_n) * \tan(x_n) * \ln(3) * \sinh(x_n) \\
 & - 3 * \sec(x_n) * \tan(x_n) * \ln(3) * \cos(x_n) + 5 * \cosh(x_n) * \tan(x_n) \\
 & - 5 * \sin(x_n) * \tan(x_n) + \ln(3)^2 * \sinh(x_n) + \ln(3)^2 * \cos(x_n) + 6 * \cosh(x_n) * \tan(x_n)^3 \\
 & - \sinh(x_n) + \cos(x_n) - 6 * \sin(x_n) * \tan(x_n)^3 - 2 * \tan(x_n)^2 * \sinh(x_n) + 2 * \tan(x_n)^2 * \cos(x_n))
 \end{aligned}$$

### 3. IMPLEMENTATION OF THE METHOD

Step size  $h = 0.001$

#### Problem 1

Consider the initial value problem  $y' = \sin(5x) - 0.4y$ ,  $y(0) = 5$  with theoretical solution

$$y(x) = \frac{(3270 * \exp(-(2*x)/5))}{629} - \frac{(125 * \cos(5*x))}{629} + \frac{(10 * \sin(5*x))}{629}$$

step size  $h = 0.1$

<i>n</i>	<i>x</i>	LETH Scheme	Theoretical	Error
1.0	0.0	4.980539e+00	4.980290e+00	2.491870e-04
2.0	0.0	4.961653e+00	4.961156e+00	4.966522e-04
3.0	0.0	4.943337e+00	4.942595e+00	7.417817e-04
4.0	0.0	4.925586e+00	4.924602e+00	9.839672e-04
5.0	0.1	4.908392e+00	4.907170e+00	1.222608e-03
6.0	0.1	4.891747e+00	4.890290e+00	1.457111e-03
7.0	0.1	4.875641e+00	4.873954e+00	1.686895e-03
8.0	0.1	4.860064e+00	4.858153e+00	1.911391e-03
9.0	0.1	4.845004e+00	4.842874e+00	2.130040e-03
10.0	0.1	4.830447e+00	4.828105e+00	2.342302e-03

<i>n</i>	<i>x</i>	LETH Scheme	Theoretical	Error
1.0	0.0	4.998005e+00	4.998003e+00	2.499319e-06
2.0	0.0	4.996017e+00	4.996012e+00	4.997565e-06
3.0	0.0	4.994034e+00	4.994026e+00	7.494677e-06
4.0	0.0	4.992056e+00	4.992046e+00	9.990592e-06
5.0	0.0	4.990085e+00	4.990072e+00	1.248525e-05
6.0	0.0	4.988119e+00	4.988104e+00	1.497859e-05
7.0	0.0	4.986159e+00	4.986142e+00	1.747054e-05
8.0	0.0	4.984205e+00	4.984185e+00	1.996105e-05
9.0	0.0	4.982257e+00	4.982235e+00	2.245005e-05
10.0	0.0	4.980314e+00	4.980290e+00	2.493749e-05

#### Problem 2

$y' = -10 * (y - 1)^2$ ,  $y(0) = 2$  and

$$y(x) = 2 - x * (10 * y^2 - 20 * y + 10);$$

in interval [0,0.2], h is tried at 0.1, 0.01

Step size h = 0.01

n	x	LETH Scheme	Theoretical	Error
1.0	0.0	1.909001e+00	1.909091e+00	8.948137e-05
2.0	0.0	1.833202e+00	1.833333e+00	1.309066e-04
3.0	0.0	1.769083e+00	1.769231e+00	1.476357e-04
4.0	0.0	1.714134e+00	1.714286e+00	1.514653e-04
5.0	0.1	1.666518e+00	1.666667e+00	1.485826e-04
6.0	0.1	1.624858e+00	1.625000e+00	1.423228e-04
7.0	0.1	1.588101e+00	1.588235e+00	1.345150e-04
8.0	0.1	1.555429e+00	1.555556e+00	1.261686e-04
9.0	0.1	1.526198e+00	1.526316e+00	1.178363e-04
10.0	0.1	1.499890e+00	1.500000e+00	1.098123e-04

$$y(x)=\exp(-10*x)$$

Step size h = 0.01

n	x	LETH Scheme	Theoretical	Error
1.0	0.0	9.048338e-01	9.048374e-01	3.596574e-06
2.0	0.0	8.187243e-01	8.187308e-01	6.472776e-06
3.0	0.0	7.408095e-01	7.408182e-01	8.734542e-06
4.0	0.0	6.703096e-01	6.703200e-01	1.047394e-05
5.0	0.1	6.065189e-01	6.065307e-01	1.177089e-05
6.0	0.1	5.487989e-01	5.488116e-01	1.269464e-05
7.0	0.1	4.965720e-01	4.965853e-01	1.330512e-05
8.0	0.1	4.493153e-01	4.493290e-01	1.365410e-05
9.0	0.1	4.065559e-01	4.065697e-01	1.378620e-05
10.0	0.1	3.678657e-01	3.678794e-01	1.373977e-05

Step size h = 0.001

n	x	LETH Scheme	Theoretical	Error
1.0	0.0	1.990099e+00	1.990099e+00	9.777569e-09
2.0	0.0	1.980392e+00	1.980392e+00	1.888578e-08
3.0	0.0	1.970874e+00	1.970874e+00	2.737086e-08
4.0	0.0	1.961538e+00	1.961538e+00	3.527224e-08
5.0	0.0	1.952381e+00	1.952381e+00	4.262955e-08
6.0	0.0	1.943396e+00	1.943396e+00	4.947587e-08
7.0	0.0	1.934579e+00	1.934579e+00	5.584715e-08
8.0	0.0	1.925926e+00	1.925926e+00	6.177164e-08
9.0	0.0	1.917431e+00	1.917431e+00	6.727944e-08
10.0	0.0	1.909091e+00	1.909091e+00	7.239604e-08

With step size h = 0.001

n	x	LETH Scheme	Theoretical	Error
1.0	0.0	9.900498e-01	9.900498e-01	3.704350e-10
2.0	0.0	9.801987e-01	9.801987e-01	7.331288e-10
3.0	0.0	9.704455e-01	9.704455e-01	1.088232e-09
4.0	0.0	9.607894e-01	9.607894e-01	1.435779e-09
5.0	0.0	9.512294e-01	9.512294e-01	1.775977e-09
6.0	0.0	9.417645e-01	9.417645e-01	2.108897e-09
7.0	0.0	9.323938e-01	9.323938e-01	2.434678e-09
8.0	0.0	9.231163e-01	9.231163e-01	2.753379e-09
9.0	0.0	9.139312e-01	9.139312e-01	3.065182e-09
10.0	0.0	9.048374e-01	9.048374e-01	3.370138e-09

Problem 3:

$$y' = \exp(10(x-y));, y(0) = 0.1 \text{ and}$$

$y(x) = \log(\exp(10x) + \exp(1)-1)/10$  in interval [0,1], h is tried at 0.1, 0.01

Step size h = 0.001

n	x	LETH Scheme	Theoretical	Error
1.0	0.0	1.003672e-01	1.000000e-01	3.672044e-04
2.0	0.0	1.007367e-01	1.003690e-01	3.676924e-04
3.0	0.0	1.011086e-01	1.007404e-01	3.681816e-04
4.0	0.0	1.014828e-01	1.011141e-01	3.686721e-04
5.0	0.0	1.018593e-01	1.014902e-01	3.691638e-04
6.0	0.0	1.022382e-01	1.018686e-01	3.696567e-04
7.0	0.0	1.026195e-01	1.022494e-01	3.701508e-04
8.0	0.0	1.030031e-01	1.026325e-01	3.706460e-04
9.0	0.0	1.033891e-01	1.030180e-01	3.711424e-04
10.0	0.0	1.037775e-01	1.034058e-01	3.716400e-04

Problem 5:

$$y' = y^2, y(0) = 1, \text{ which have theoretical solution}$$

$y(x) = \frac{1}{1-x}$  in the interval  $0 \leq x \leq 1$ . We solve at step size of h = 0.01 and 0.001

Step size h = 0.01

n	x	LETH Scheme	Theoretical	Error
1.0	0.0	1.010101e+00	1.010101e+00	7.080961e-09
2.0	0.0	1.020408e+00	1.020408e+00	1.483549e-08
3.0	0.0	1.030928e+00	1.030928e+00	2.331732e-08
4.0	0.0	1.041667e+00	1.041667e+00	3.258586e-08
5.0	0.1	1.052632e+00	1.052632e+00	4.270682e-08
6.0	0.1	1.063830e+00	1.063830e+00	5.375311e-08
7.0	0.1	1.075269e+00	1.075269e+00	6.580579e-08
8.0	0.1	1.086956e+00	1.086957e+00	7.895520e-08
9.0	0.1	1.098901e+00	1.098901e+00	9.330234e-08
10.0	0.1	1.111111e+00	1.111111e+00	1.089604e-07

Problem 4:

$$y' = -10 * y, y(0) = 1$$

Step size h = 0.001

n	x	LETH Scheme	Theoretical	Error

1.0	0.0	1.001001e+00	1.001001e+00	6.885603e-13
2.0	0.0	1.002004e+00	1.002004e+00	1.383116e-12
3.0	0.0	1.003009e+00	1.003009e+00	2.084555e-12
4.0	0.0	1.004016e+00	1.004016e+00	2.792211e-12
5.0	0.0	1.005025e+00	1.005025e+00	3.506528e-12
6.0	0.0	1.006036e+00	1.006036e+00	4.227507e-12
7.0	0.0	1.007049e+00	1.007049e+00	4.955147e-12
8.0	0.0	1.008065e+00	1.008065e+00	5.689227e-12
9.0	0.0	1.009082e+00	1.009082e+00	6.430412e-12
10.0	0.0	1.010101e+00	1.010101e+00	7.178480e-12

#### 4. CONCLUSION

We have been able to develop a numerical method which is composed of logarithm, exponential, trigonometric and hyperbolic basis functions; we have also solved some sampled initial value problems in ordinary differential equations numerically using this new method. The method performed well on the problems and can be said to be numerically stable.

#### 5. REFERENCES

- Fatunla, S. O. (1976). "A New Algorithm for Numerical Solution of Ordinary Differential Equations". *Computer and Mathematics with Applications*, 2:247-253.
- Fatunla, S. O. (1988). "Numerical Methods for Initial Value Problems in Ordinary Differential Equations". Academic Press Inc, New York.

Henrici, P. (1962). "Discrete Variable Methods in Ordinary Differential Equations". John Wiley and Sons, New York.

Lambert, J. D. (1973). "Computational Methods in Ordinary Differential Equations". John Wiley and Sons, New York.

Lambert, J. D. (1991). "Numerical Methods for Ordinary Differential Systems: The Initial Value Problem". John Wiley and Sons, New York.

Ibijola, E.A., Bamisile, O.O. and Sunday, J. (2011). On the derivation and applications of a new one-step method based on the combination of two interpolating functions, *American Journal of Scientific and Industrial Research*, 2, 422-427.

Odekunle, M. R., Adesanya, A. O. and Sunday, J. (2012) . 4-Point block method for direct integration offirst-order ordinary differential equations, *International Journal of Engineering Research and Applications*, 2,1182-1187.

Rosser, J. B. (1967). A Runge-Kutta method for all Seasons, *SIAM Review*. 9, 417-452.